

# Carnap's Philosophy of Mathematics

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## Abstract

For several decades Carnap's philosophy of mathematics used to be either dismissed or ignored. It was perceived as a form of linguistic conventionalism and thus taken to rely on the bankrupt notion of truth by convention. However, recent scholarship has revealed a more subtle picture. It has been forcefully argued that Carnap is not a linguistic conventionalist in any straightforward sense, and that apparently decisive objections to his position target a straw man. This raises two questions. First, how exactly should we characterise Carnap's actual philosophy of mathematics? Secondly, is his position an attractive alternative to established views? I will tackle these by looking at Carnap's response to the incompleteness theorems. Drawing on objections put forward by Gödel and Beth, I argue that some crucial aspects of Carnap's positive account have remained underdeveloped. Suggestions on what a full evaluation of Carnap's position requires are made.

## 1 Introduction

Unlike Frege, Hilbert, or Gödel, Carnap does not usually play a central role in textbooks or introductory courses on the philosophy of mathematics. Not because he didn't have anything to say about this topic: in his 1934 book *The Logical Syntax of Language*, Carnap puts forward an approach to the philosophy of mathematics that is both unique and interesting.<sup>1</sup> But its reception has been troubled. For several decades Carnap was widely read as a linguistic conventionalist about mathematical truth, and thus taken to be refuted by the anti-conventionalist arguments of Quine and others. Recent scholarship on Carnap has challenged this picture and shown that there is no easy refutation of his position. But the relevant literature is not always easily accessible and tends to be read by historians of analytic philosophy only. Misconceptions about Carnap's project therefore remain widespread.

In this paper I will first clarify the descriptive question of what Carnap's position actually is. It will be helpful to proceed by asking whether the principle of

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<sup>1</sup> I will not cover Carnap's early philosophy of mathematics. See Awodey and Carus 2001 and Schiemer 2013.

tolerance and the analytic/synthetic distinction commit him to linguistic conventionalism. We can then move on to the evaluative question of whether Carnap's account is stable and attractive. I will discuss the two most sophisticated objections that have been put forward against it, both of which are based on the incompleteness theorems: Gödel's consistency argument and Beth's argument from non-standard models. Whether these are ultimately successful cannot be conclusively settled here. I will argue, however, that friends of Carnap's position cannot just ignore the arguments as being based on misreadings. Suggestions for future research are made.

## 2 Carnap and Mathematics

### 2.1 Tolerance and Analyticity

It would be convenient to characterise Carnap's philosophy of mathematics by relating it to more established views in the field, such as Platonism, formalism, or nominalism. But this is difficult, since Carnap's mature approach to philosophy differs quite drastically from that of typical proponents of such views. We can start by noticing that Carnap is happy to *talk* like a Platonist. His well-known "Empiricism, Semantics, and Ontology" argues that empiricists can use languages that refer to abstract objects. Carnap thus makes Platonist-sounding claims such as

- Numbers exist (Carnap 1956a: 208).
- Numerals refer to numbers (Carnap 1956a: 216).
- Numbers are mind-independent entities.<sup>2</sup>

The acceptance of abstract objects is usually thought to face various philosophical challenges. Here is a representative collection:

- **Ontology:** Are abstract objects compatible with a naturalistic worldview? Are they indispensable?
- **Philosophy of language:** How is reference to abstract objects possible? Causal theories of reference don't work; so what's the alternative?

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<sup>2</sup> Technically he only says this about propositions, but the consideration put forward applies to all abstract objects (Carnap 1956a: 210f).

- **Epistemology:** How can we know about abstract objects? Causal theories of knowledge don't work; so what's the alternative?

Carnap, however, doesn't really address these questions directly, at least not in a way that resembles mainstream discussions. In order to understand this lack of engagement we need to introduce two core components of his position: the principle of tolerance and the notion of analyticity.

The principle of tolerance is the key innovation of *The Logical Syntax of Language*. Carnap claims that, thus far, debates in the philosophy of logic and mathematics have suffered from a flawed presupposition: that one needs to *justify* the acceptance of a logic using philosophical arguments.<sup>3</sup> However, he says this is unnecessary. According to Carnap, adopting a logic merely requires that clear rules for its use have been laid down. Philosophers are thus encouraged to "give syntactical rules instead of philosophical arguments" (Carnap 1937a: 52). Freed from the requirement of philosophical justification, we can avoid "pseudo-problems and wearisome controversies" in favour of a "boundless ocean of unlimited possibilities" (Carnap 1937a: xv).<sup>4</sup>

The principle of tolerance partly explains why Carnap thinks that we can use a language that quantifies over mathematical objects without addressing traditional philosophical questions. For him, the acceptance of such a language does not require a prior "ontological insight" (Carnap 1956a: 214). But the principle is not sufficient in itself. Carnap is an empiricist, after all, and empiricists typically hold that cognitively meaningful statements have to be related to statements about observations. This is not the case for discourse about numbers and other abstract objects. So in order to properly accommodate them, Carnap relies on the analytic/synthetic distinction.

When talking about languages, Carnap usually refers to formalised languages with explicitly stated rules of uses. (Later he would call these *linguistic frameworks*.) In such a language some sentences are derivable from the rules itself, just as logical truths can be derived in a calculus without any additional premises. Sentences that follow from the linguistic rules are classified as *analytic*, whereas sentences that are independent of the rules themselves are *synthetic*.<sup>5</sup> Empirical claims about the observable world are the paradigmatic examples of synthetic statements (Carnap 1937a: 41). But analytic sentences are meaningful too, despite being inferentially isolated from statements about observations (Carnap

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<sup>3</sup> In *Logical Syntax* Carnap construes logic broadly to also include mathematics.

<sup>4</sup> The question of how – if at all – Carnap justifies the principle of tolerance *itself* is a deep and difficult one that needs to be set aside here. See Putnam 1983 and Ricketts 1994.

<sup>5</sup> A sentence whose negation is analytic is *contradictory*.

1937a: 318f). And, importantly, Carnap sets up the languages he constructs in such a way that mathematics comes out as analytic. Therefore, he thinks, no philosophical obstacles stand in the way of using mathematical language.

## 2.2 Is this Conventionalism?

Carnap was and is frequently read as a conventionalist about mathematical truth (Putnam 1979, Potter 2000: chapter 11, Warren 2020: chapter 13). And this is not surprising. Linguistic conventionalism says, roughly, that the truth of mathematical statements is somehow explained by linguistic conventions. Carnap holds that mathematical statements are analytic, which means that they follow from the rules of a certain constructed language. Furthermore, Carnap thinks that adopting such rules does not require any prior justification, but can proceed on purely pragmatic grounds. Taken together, these two ideas seem to amount to the conventionalist thesis.

It is then also not surprising that Carnap's philosophy of mathematics has been widely dismissed. For there are two powerful arguments that are often seen as decisive refutations of conventionalism: Quine's circularity objection (Quine 1949) and the master argument against the very idea of truth by convention popularised by Boghossian (Boghossian 2017). Let us therefore consider whether an easy refutation of Carnap along anti-conventionalist lines is possible.

In his influential "Truth by Convention", Quine argues against conventionalism about logic. The key charge is one of circularity. Since there are infinitely many logical truths, conventions would somehow need to settle the truth values of infinitely many sentences. But, on the other hand, the conventions need to be storable in a finite (or at least recursive) way.<sup>6</sup> Infinitely many stipulated truths thus only arise if some of the conventions are general in nature, such as

Let every instance of the following schema be true:  $\lceil \phi \rightarrow \phi \rceil$ .

But in order to show that, say,  $\lceil q \rightarrow q \rceil$  is such an instance, logical rules are needed – in particular universal instantiation and modus ponens. The relevant conventions thus *presuppose* principles of logic that they do not account for. Conventionalism, says Quine, is viciously circular.

This may be a powerful argument against *some* versions of conventionalism. But is it any good against Carnap? No. As scholars have stressed, it is implausible to read Carnap as attempting to give a *non-circular* explanation of logical

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<sup>6</sup> This requirement can be challenged by distinguishing between implicit and explicit conventions (Warren 2017).

(and mathematical) truth in the sense Quine has in mind (Goldfarb and Ricketts 1992: 71, Goldfarb 1995: 330). Carnap is very open and upfront about the fact that his definitions of 'analytic' for the mathematical object language can only be given in a metalanguage that is itself mathematical. Indeed, in many cases the metalanguage must even be stronger than the object language (Carnap 1937a: 100, 129). It is thus not credible that Carnap was unaware of the point Quine made. The better explanation is rather that he did not aim for non-circularity in the first place.<sup>7</sup>

Quine's argument relies on the fact that there are infinitely many logical truths. A second influential argument against conventionalism is more general and challenges the very idea that conventions can make *any* sentence true. The historical origins of this master argument are unclear, but it was popularised by Boghossian in the 1990s.<sup>8</sup> Consider how conventions could influence the truth of a sentence. It is clear that they somehow determine what a sentence, understood purely syntactically, means. And since the meaning of a sentence determines its truth conditions, everyone agrees that conventions play *some* role in determining truth. The conventionalist thesis about mathematics, however, must be something stronger than this. The natural reading of it goes as follows:

Whereas conventions always determine which proposition a sentence expresses, in the case of logic and mathematics they also *make it the case* that the proposition expressed is true.

Boghossian claims that this is absurd, since it would have been true that snow is either white or it isn't even if there had been no conventions at all (Boghossian 2017: 583).<sup>9</sup>

This may be a powerful argument against *some* versions of conventionalism. But is it any good against Carnap? No. While some of his formulations sound like the conventionalist thesis above, scholars have argued that they should not be read in a metaphysically loaded way: Carnap "rejects any thick notion of truth-in-virtue-of" (Ricketts 2007: 211, see also Ebbs 2017: 25). Some of Carnap's frameworks allow us to talk about propositions. But in none of them does he draw a distinction between the ways in which these propositions are made true. This is a notion Carnap has no use for. So whatever the analyticity of mathematics is supposed to amount to, a Boghossian-style reconstruction – like the Quinean critique – needs to be rejected as uncharitable.

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<sup>7</sup> Did Quine *intend* to target Carnap with his critique? See Ebbs 2011 and Morris 2018 for cases against this.

<sup>8</sup> Yablo ascribes it to Casimir Lewy (Yablo 1992: 878).

<sup>9</sup> Carnap agrees (Carnap 1937b: 37f).

### 2.3 Quietism and Voluntarism

We have seen that there is no quick and easy way to refute Carnap's position by relying on established anti-conventionalist arguments. But the discussion so far has been curiously negative. We learned that Carnap does *not* strive to avoid circularity and has *no* room for the notion of truth-in-virtue-of. But can one also characterise the nature of his account in a more positive way?

That it is difficult to pin Carnap's position down demonstrates the quietist and voluntarist strains in his conception of philosophy. It is quietist in the sense that Carnap leaves no room to even *formulate* certain questions that many others take to be crucial (Goldfarb 1997: 61). For this reason some philosophical puzzles about mathematics are not resolved but rather dissolved – or, as a critic might say, ignored. Carnap's conception is voluntarist in the sense that, unusually for a theoretical philosopher, he does not see himself as putting forward descriptive theses (Jeffrey 1994). When he writes that mathematics is analytic, for instance, this is primarily a *proposal* to adopt a language with rules that classify mathematics as analytic. This unique way of thinking about the philosophy of mathematics does not easily fit into established classifications.

Is Carnap's position so idiosyncratic that fruitful interactions between it and mainstream philosophy of mathematics are impossible?<sup>10</sup> I think not, but it is difficult to see this at such a high level of abstraction. I will therefore discuss two specific objections to Carnap's position that cannot be set aside as easily as those in section 2.2. These are Gödel's consistency argument and Beth's argument from non-standard models. Going through them in the next two sections, and surveying the responses suggested on Carnap's behalf, will demonstrate that not anything goes. There are problems that need addressing, whether fatal or not. We will thus gain a better sense of the positive features of Carnap's position.

Both arguments rely on Gödel's incompleteness theorems. These theorems establish that there is no theory T which has all of the following properties:

- (1) T is consistent
- (2) T is recursively formalised<sup>11</sup>
- (3) T is strong enough to do basic arithmetic

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<sup>10</sup> Putnam seems worried about this when writing that "the Carnap who holds no doctrines but only asks for "clarification" [...] is just not the Carnap I knew and loved" (Putnam 1994: 281).

<sup>11</sup> This means that T's axioms can be recursively enumerated and that its inference rules are decidable (Raatikainen 2020: section 1.1).

- (4) T is complete, i.e. for any T-sentence  $\phi$ , either T proves  $\phi$  or T proves  $\neg\phi$

Gödel's theorems are commonly taken to refute at least one position in the philosophy of mathematics, namely Hilbertian formalism. And Gödel himself thought that they also pose a serious challenge to Carnap's position.

### 3 Gödel's Consistency Argument

#### 3.1 Incompleteness and Consistency

In 1953 Gödel was asked to contribute to the Schilpp volume on Carnap. Over the coming years he produced a number of drafts with the title "Is Mathematics Syntax of Language?". In 1959 Gödel decided not to have any of them published, however, so Carnap himself never read the critique. I will focus on the most important and widely discussed argument present in the drafts: the argument from consistency.<sup>12</sup>

Gödel's second incompleteness theorem shows that no consistent and sufficiently strong mathematical theory proves its own consistency. The consistency of Peano arithmetic (PA), for instance, cannot be demonstrated in PA itself, or in any weaker theory. Stronger mathematics is needed, such as set theory. Gödel thought that this result affects Carnap's position, which he took to be that "[m]athematical intuition, for all scientifically relevant purposes, [...] can be replaced by conventions about the use of symbols and their application" (Gödel 1995: 356).

In order to concisely state Gödel's argument it will be helpful to rely on the familiar notion of a *conservative extension*:

Theory  $T^*$  is a conservative extension of a theory  $T$  iff

- (i) every theorem of  $T$  is also a theorem of  $T^*$  and
- (ii) every theorem of  $T^*$  that is expressed in the language of  $T$  is also a theorem of  $T$ .

Suppose that we currently accept a (consistent) base theory in which we can make claims about the empirical world, such as "this table is black", but which does not yet contain mathematics. Carnap thinks that, if we want to, we could

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<sup>12</sup> See Lavers 2019 for other considerations Gödel puts forward.

extend this theory by adding linguistic conventions for the use of mathematical symbols. Gödel challenges this assumption in the following way:

- (1) If some theory is adopted as a convention, it must be known that it is a conservative extension of the base theory.
- (2) A theory that extends a consistent base theory is conservative only if the former theory is consistent.
- (3) So: A mathematical theory can be adopted as a convention only if it is known that this theory is consistent.
- (4) The incompleteness theorems show that, for any sufficiently strong mathematical theory, we need even stronger mathematics to prove that theory's consistency.
- (5) So we need to rely on mathematical intuition at some point in order to know that the theory we want to adopt as a convention is consistent.

One contentious point in this argument is the step from (4) to (5). What does Gödel mean by mathematical intuition? And how exactly is it connected to the need for stronger mathematics? But discussing this part of the argument would lead us to far afield, since those who defend Carnap have focused their attention on premise (1).<sup>13</sup> Let us therefore consider why Gödel accepted it.

The motivation behind (1) is a natural one. We assumed that, in the base theory, we can talk about empirical objects. If we extend it by a non-conservative theory, then new empirical claims become derivable. In particular, if we extend it by an inconsistent theory, *every* sentence is derivable, including "there are seven billion red tables in Iceland". This is a false empirical claim. And this shows, so Gödel, that the addition of the mathematical theory cannot be classified as conventional. Conventionality requires that nothing can *refute* or falsify the added theory. Otherwise we are dealing with a *hypothesis* rather than a convention (Gödel 1995: 339). It is therefore crucial to insist on the conservativeness of mathematics, which in turn requires a consistency proof.

### 3.2 Facts and Conventions

The canonical reply to Gödel's argument has been given by Goldfarb and Ricketts. According to them, Gödel's objection misses its target, since it presupposes

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<sup>13</sup> For more on Gödelian intuition see Potter 2001 and Wrigley forthcoming.

a "a language-transcendent notion of empirical fact" rejected by Carnap (Goldfarb and Ricketts 1992: 65). But it is hardly transparent what this means (Eklund 2012). So let us go through their defence using the previous example.

We supposed that the sentence

(T) There are seven billion red tables in Iceland

is expressible in, but not entailed by, the empirical base theory. Adding inconsistent mathematics then makes every sentence derivable, including (T). Gödel took this to show that violating the consistency requirement leads to false empirical claims.

Goldfarb and Ricketts want to resist the latter conclusion. They stress that, while the *sentence* (T) indeed becomes derivable in the inconsistent theory, we should *not* take it to make any empirical claim at all. This is because Carnap accepts a form of holism on which the empirical content of sentences is determined by the theory they are part of (Carnap 1937a: 175). In an inconsistent theory, no sentence has any empirical content since they all have the same trivial inferential profile. This point can also be put in terms of the distinction between sentences and the propositions they express. Whereas (T), considered as part of the base theory, expresses an empirical proposition, every sentence of an inconsistent theory – and thus also (T) – expresses the same trivial proposition.

One can question whether this move is really a *defence* of Carnap, however, rather than the illustration of an even more serious problem. Potter has argued for the latter conclusion, by pointing out the following consequences of Carnap's position:

- (1) According to Carnap's holism only consistent languages make any claims about the world.
- (2) The language we use includes mathematics.
- (3) We have no a priori proof of the consistency of our language.
- (4) We therefore have at most inductive empirical evidence that our language is consistent.
- (4) Consequently we have at most inductive empirical evidence that the sentences of our language make any claims about the world.

Potter describes this conclusion as "plainly absurd", since "Carnap's view makes it an experimental fact that I have a conception of an empirical world at all" (Potter 2000: 277). One might quibble whether so strong an indictment is justified. But there is certainly reason to be concerned about Carnap's holistic conception of how language relates to reality.

### 3.3 Frameworks and Language

Potter's improved version of Gödel's consistency argument has not been left unchallenged by those sympathetic to Carnap. In their "How Carnap Could Have Replied to Gödel", Awodey and Carus highlight Carnap's distinction between *formal languages* with explicit rules and informal and intuitive *natural language*:

Though [Carnap's] conception of the scientific, theoretical language was holistic [...], he specifically saw the practical, intuitive part of language as distinct from it and serving a different purpose. (Awodey and Carus 2004: 213)

According to Awodey and Carus, the apparently absurd conclusion Potter drew is avoided because "the purely descriptive sentences [of ordinary language] remain unaffected by an inconsistency in the theoretical language [...] since the interpretation of this language is given by practical agreement in its use" (Awodey and Carus 2004: 214n19). Here they follow Carnap's own remark that the language we use to talk about observable things has a "complete interpretation" because it "is used by a certain language community as a means of communication, and [...] all sentences of [it] are understood by all members of the group in the same sense" (Carnap 1956b: 40).

What are we to make of this reply? On the one hand, Awodey and Carus are right to stress that the distinction between formalised frameworks and the intuitive natural language is important for Carnap. It is also true that neither Gödel nor Potter are sufficiently sensitive to this distinction in their exegesis. But, on the other hand, it is hard to be fully satisfied, since we learn too little about how exactly our use of language manages to give it an interpretation that enables it to describe an objective world. In his mature work Carnap tends to take the observation language for granted and focuses his attention on the development of frameworks for the theoretical part of language.<sup>14</sup> A full response to the Gödel-Potter argument, however, arguably requires a more developed story of how language makes contact with reality in a non-holistic way.

Neither Gödel's original consistency argument nor Potter's improved version has delivered a straightforward refutation of Carnap's position. But nor do the defensive moves allay all concerns. A convincing defence should have more to say on how the way we use language forges the connection between formalised

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<sup>14</sup> Quine, on the other hand, starts to tackle the issue of objectivity in the 1950s (Quine 1957, Quine 1960).

systems and the world. That the issue of language use is crucial for understanding and evaluating Carnap's position is further supported by the analysis of Beth's objection we will turn to now.

## 4 Beth's Argument from Non-Standard Models

### 4.1 Incompleteness Again

A paper by Beth that was actually published in the Schilpp volume contains a second argument based on the incompleteness theorems. In order to understand Beth's basic point we first need to take a closer look at the definitions of 'analytic' Carnap gives in *Logical Syntax*. His aim is to classify every true mathematical sentence as analytic and every false one as contradictory. In other words, the definition of 'analytic' is supposed to be *complete*, leaving the status of no purely mathematical sentence undetermined. But how, in light of Gödel's theorems, is this even possible?

The task would be hopeless if Carnap wanted to define analyticity in terms of what is derivable from a recursively formalised theory. But Gödel's theorems don't apply to *non-recursive* theories, and Carnap makes use of this fact. For language I of *Logical Syntax* he relies on the infinitary  $\omega$ -rule (Carnap 1937a: 173):

$$\begin{array}{c} \omega\text{-RULE} \\ \frac{\phi(0), \phi(1), \phi(2), \dots}{\forall x\phi(x)} \end{array}$$

For language II Carnap's definition is effectively a version of a Tarskian truth-theory, even though it is still presented as a syntactic definition (Coffa 1987, Koellner 2009). In particular, Carnap characterises analyticity for language II by relying on a domain of quantification containing the accented expressions  $0, 0', 0'', \dots$  (Carnap 1937a: 106). These are assumed to be isomorphic to the natural numbers, and so analyticity is equated with truth in the standard model of arithmetic ( $\mathbb{N}$ ). In modern terminology Carnap's definition thus amounts to the following:

$$\phi \text{ is analytic in language II iff } \mathbb{N} \models \phi.$$

Beth takes issue with Carnap's reliance on an infinite stock of accented expressions. After rightly stressing that these expressions need to correspond to the

standard numerals, Beth points out – also rightly – that enumerating some instances does not suffice to pin down the intended interpretation. Beth illustrates this by introducing the fictitious logician Carnap\*, who thinks that there are *more* accented expressions than standard numerals, and therefore equates analyticity in language II with truth in a *non-standard* model of arithmetic. Based on this non-standard interpretation Carnap\* then challenges some of Carnap's results, since they presuppose the standard interpretation (Beth 1963: 480f).

Beth clearly thought that the possibility of someone like Carnap\* is a problem for Carnap's position. But his paper is rather elusive about what exactly this problem is supposed to be. In the secondary literature, two readings of Beth can be distinguished: one puts the emphasis on the apparent *model-theoretic* character of the argument, while the other draws a closer connection to considerations about *rule-following*. We will discuss these in turn.

## 4.2 The Skolemite Reading

After introducing Carnap\*, Beth writes that "the above considerations [...] are only variants of the Löwenheim-Skolem paradox" (Beth 1963: 478). Ricketts takes this allusion very seriously. One interesting consequence of the Löwenheim-Skolem theorem is that Zermelo-Fraenkel set theory (ZFC) can be interpreted in a *countable* model. Skolem thought that there was something paradoxical about this, since ZFC proves that there is an uncountable set.<sup>15</sup> According to Ricketts, Beth's point is that the same observation applies to the language in which *Logical Syntax* is written itself – call this the *syntax language*. In the case of ZFC the intended interpretation is uncountable and the non-standard interpretation countable. In the case of the syntax language the situation is reversed. The intended interpretation is isomorphic to the natural numbers and hence countable, whereas the non-standard interpretation is uncountable.

If this is Beth's argument, however, then Carnap should not be overly concerned about it. For what is so problematic about the availability of non-standard interpretations in itself? Some have taken Skolem's observation to imply that ZFC cannot "really" talk about uncountable sets, but such interpretations are widely rejected (Tymoczko 1989). In the case of the syntax language, moreover, it is difficult to discern even the *appearance* of a paradox brought about by the possibility of interpreting it in an uncountable model. One idea could be that the syntax language is not "really about syntax" unless one pins down a unique

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<sup>15</sup> See Bays 2014 for the technical and philosophical background.

intended interpretation. But it hard to see why Carnap should agree to an assumption like that.

However, there is reason to think that Ricketts' model-theoretic reconstruction does not capture the argument Beth had in mind. Ricketts for instance writes that the divergence between Carnap and Carnap\* "need not be manifest in their use of the sentences of the informal syntax language" (Ricketts 2004: 194). But in Beth's own example they *do* disagree in this way, namely about the consistency sentence for language II (Beth 1963: 478). Furthermore, Ricketts' Skolemite reading of Beth does not make any use of the incompleteness theorems. But in Beth's own example they do play an essential role. They are required to prove the existence of the non-standard model the example relies on (Beth 1963: 477). In his own reply to Beth (Carnap 1963: section 18), Carnap likewise ignores the role of incompleteness and thus leaves central parts of the argument unaddressed (Ben-Menahem 2006: 209).

Beth himself introduced the comparison to Skolem's paradox. But the analogy does not seem to fit the way his actual reasoning proceeds. The interesting question is thus whether an alternative reading of the argument can make it more forceful. One attempt to do so is the rule-following interpretation of Beth.

### 4.3 The Rule-Following Reading

Beth writes that the case of Carnap\* demonstrates a "limitation regarding the Principle of Tolerance" (Beth 1963: 479). Given how Beth reasons, it is plausible that the alleged limitation has something to do with the fact that Carnap uses non-recursive methods to define analyticity. Friedman has tried to spell out an objection along these lines in a number of ways (Friedman 1999a, Friedman 1999b). But in response to criticism he has come round to the view that Beth's objection is ineffective (Friedman 2009).

A new interpretation of Beth that follows the earlier Friedman in taking the objection to be forceful has recently been put forward by Marschall. On this reading Beth's point is more akin to the rule-following arguments of Wittgenstein and Kripke than the model-theoretic arguments of Skolem and Putnam (Marschall 2021).<sup>16</sup> Remember that, on Carnap's voluntaristic outlook, languages I and II of *Logical Syntax* are intended as proposals for adoption. On the rule-following interpretation, Beth's point is to question whether the rules Carnap uses to define 'analytic' in these languages can be adopted in practice.

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<sup>16</sup> See Ebbs 1997: sections 60-61 for a similar interpretation that, however, makes no use of infinitary rules.

It is easiest to appreciate the potential problem for language I, in which Carnap relies on the infinitary  $\omega$ -rule. He writes that "there is nothing to prevent the practical application of such a rule" (Carnap 1937a: 173). But the prevailing consensus in the philosophy of mathematics denies this, and takes only recursive inference rules to be practically usable (McGee 1991, Field 1994, Raatikainen 2005, Button and Walsh 2018: chapter 7).

The situation may seem better for language II and its semantic definition of analyticity. But this is illusory. As Beth has demonstrated, this definition crucially depends on assumptions about what the domain of quantification contains. Carnap thinks that "one of the semantic rules [of language II] specifies the universe of discourse" (Frost-Arnold 2013: 77). But without further clarification it is not clear what it even means for rules to *specify* a universe of discourse. We might want to say that Carnap's non-standard interpretation of the syntax language uses an *incorrect* universe of discourse. But in what sense and by what means did Carnap specify the universe of discourse to only contain standard numerals? His own examples of linguistic rules that are adopted concern much more straightforward cases, such as talking about temperature (Carnap 1950: 9f). This does not suffice to illuminate the case of analyticity.

Two kinds of responses to this version of Beth's argument are possible. One is going on the offensive and claiming that there is no problem about adopting the rules Carnap gives for analyticity. For the special case of the  $\omega$ -rule one can draw on Warren's recent work which challenges the conventional wisdom about the rule's alleged unusability (Warren 2020, Warren forthcoming). The second kind of response denies the underlying assumption that we need to adopt the rules for analyticity in any substantial sense. This is suggested by Ricketts at one point (Ricketts 2003: 262) and will appeal to those who stress the quietist elements in Carnap's philosophy. One open question is whether, given this route, one can make sense of the importance Carnap clearly assigns to the notion of analyticity.

## 5 Conclusion

The discussion of the arguments by Gödel and Beth demonstrates the importance of *language use* in assessing Carnap's project: it is essential for giving languages an interpretation and for adopting explications. Unfortunately this is a topic Carnap himself said relatively little about. Important work on this issue has been published in recent years (Stein 1992, Ricketts 2003, Carus 2007, Carus 2017), but its implications for Carnap's philosophy of mathematics have

not been fully explored so far. I suggest that this is the most fruitful path forward for those interested in Carnap's approach.

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