Conventionalism in Logic and Mathematics

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Lecture 2: Quine against Truth by Convention

Overview

Plan of the four lectures:

1. Conventionalism: What, why, and how?
2. **Quine against Truth by Convention**
3. Gödel on Convention and Consistency
4. Wittgenstein and Radical Conventionalism

Quine

W.V. Quine, *Truth by Convention* (1936)

Structure

Motivations

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- It is often read as a critique of Carnap's position, but whether he is actually the intended target is controversial (Ebbs 2011).
- We will go through Quine's argument in some detail, and think about how a conventionalist could respond.

Definitions

Quine starts by talking about definitions:

> *A definition, strictly, is a convention of notational abbreviation. [...] From a formal standpoint the signs thus introduced are* **wholly arbitrary***; all that is required of a definition is that it be theoretically im- material, i.e., that the shorthand which it introduces admit in every case of unambiguous elimination in favor of the antecedent longhand. (Quine 1949: 251, my emphasis)*

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- Example: $tan \ldots = \frac{sin...}{cos...}$
- Why should we care about definitions though?

Definitions

Functionally a definition is not a premiss to theory, but a license for rewriting theory by putting definiens for definiendum or vice versa. By allowing such replacements a definition **transmits truth***: it allows true statements to be translated into new statements which are true by the same token. (Quine 1949: 251)*

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Example:

- Truth of logic: $\frac{sin\pi}{cos\pi} = \frac{sin\pi}{cos\pi}$
- Apply the contextual definition from above.
- New truth: $tan\pi = \frac{sin\pi}{cos\pi}$

Definitions

- Definitions give us a clear sense in which some truths are conventional.
- If we start from some true statements then we can generate new statements which are guaranteed to be true.
- Since definitions are arbitrary instructions on how to rewrite certain expressions they clearly deserve to be called conventional.
- So there is at least one sense of truth by convention that Quine *accepts*.

Russell's Logicism

Furthermore, Quine even grants that Russell's and Whitehead's *Principia Mathematica* successfully shows that mathematics can be reduced to logic using definitions, and is hence true by convention:

> *[... Whitehead and Russell] adopt a meager logical language as primitive, and on its basis alone they undertake to endow mathematical expressions with definitions which conform to usage in the full sense described above: definitions which not only reduce mathematical truths and falsehoods to logical ones, but reduce all statements, containing the mathematical expressions in question, to equivalent statements involving logical expressions instead of the mathematical ones. (Quine 1949: 257f)*

Relative Conventionalism

If for the moment we grant that all mathematics is thus definitionally constructible from logic, then mathematics becomes true by convention in a relative sense: mathematical truths become conventional transcriptions of logical truths. [...] **But in strictness we cannot regard mathematics as true purely by convention unless all those logical principles to which mathematics is supposed to reduce are likewise true by convention.** *(Quine 1949: 258)*

Relative Conventionalism

- We can make sense of the idea that some truths are conventional *relative* to a given set of truths.
- But full-blown conventionalists don't want to presuppose logic, but give a conventionalist account of it as well.
- Quine thinks that this cannot be done – let us see why.

Structure

Stipulation

- Goal: Logic must be true by convention in a non-relative sense.
- Quine starts by making an initially promising proposal: just *stipulate* the logical principles we need to be true.

Stipulation

[...] the alternative is open to us, on introducing a new word, of **determining its meaning absolutely** *to whatever extent we like by specifying contexts which are to be true and contexts which are to be false. [...] Since all contexts of our new word are meaningless to begin with, neither true nor false, we are free to run through the list of such contexts and pick out as true such ones as we like; those selected become* **true by fiat, by linguistic convention.** *(Quine 1949: 260)*

Stipulation

- Once again, this is relatively conventionalism-friendly.
- Quine does not seem to be worried by the completely general considerations against truth by conventions put forward by Lewy, Boghossian et al.
- (Whether this is an oversight is a good question – Sider: 'Quine's argument does not go far enough. An adequate critique must challenge the very idea of something's being "true by convention"' (Sider 2011: 100))
- But there is a specific problem about *logic* that prevents the stipulation move from doing the trick.

Stipulation

- One slight digression: Quine's remark that "all contexts of our new word are meaningless to begin with" is worth reflecting on.
- When doing formal logic we introduce new expressions such as '$\vee$', '$\neg$', and so on.
- Since they are new, we are in one sense free to specify their truth-conditions in any way we like.
- But, on the other hand, we of course want them to conform to the natural language expressions *and*, *not*, and so on.
- The latter goal constrains us: which conventions we adopt ceases to be purely arbitrary if we intend to capture certain intuitive meanings.

Problems with Logic

- The basic problem with stipulating logic to be true: there are infinitely many logical truths.
- How could we stipulate all of them to be true?
- Quine: Basically we can't.

Problems with Logic

It would appear that we sit down to a list of expressions and check off as arbitrarily true all those which, under ordinary usage, are true statements involving only our logical primitives essentially; but this picture wanes when we reflect that the number of such statements is infinite. If the convention whereby those statements are singled out as true is to be formulated in finite terms, **we must avail ourselves of conditions finite in length which determine infinite classes of expressions.** *(Quine 1949: 262f)*

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- Remember your basic logic classes: there we managed to specify the truth conditions of all sentences of TFL and FOL using finite means.

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$\boldsymbol{A} \wedge \boldsymbol{B}$ is true in an interpretation **iff**
both $\boldsymbol{A}$ is true and $\boldsymbol{B}$ is true in that interpretation

$\boldsymbol{A} \vee \boldsymbol{B}$ is true in an interpretation **iff**
either $\boldsymbol{A}$ is true or $\boldsymbol{B}$ is true in that interpretation

$\neg\boldsymbol{A}$ is true in an interpretation **iff**
$\boldsymbol{A}$ is false in that interpretation

$\boldsymbol{A} \rightarrow \boldsymbol{B}$ is true in an interpretation **iff**
either $\boldsymbol{A}$ is false or $\boldsymbol{B}$ is true in that interpretation

$\boldsymbol{A} \leftrightarrow \boldsymbol{B}$ is true in an interpretation **iff**
$\boldsymbol{A}$ has the same truth value as $\boldsymbol{B}$ in that interpretation

Problems with Logic

So why can't we stipulate something like this:

(I) Let every instance of the following schema be true: $\ulcorner\phi \rightarrow \phi\urcorner$.

(II) If a statement ϕ and a statement $\ulcorner\phi \rightarrow \psi\urcorner$ are true, then let ψ be true as well.

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> *In a word, the difficulty is that if logic is to proceed mediately from conventions, logic is needed for inferring logic from the conventions. (Quine 1949: 271)*

Problems with Logic

(I) Let every instance of the following schema be true: $\ulcorner \phi \rightarrow \phi \urcorner$.

(II) **If** a statement ϕ **and** a statement $\ulcorner \phi \rightarrow \psi \urcorner$ are true, **then** let ψ be true as well.

- If logic is true by convention, (II) must partly determine the meaning of logical expressions.
- But (II) itself contains logical terminology.
- So in order to *state* the conventions logic already needs to be presupposed.
- We thus only have a form of relative conventionalism after all.

Problems with Logic

- In the basic logic classes we assume that we have an intuitive understanding of logic in ordinary language, and we use that to define the formal languages such as TFL and FOL.
- But – so it seems – the conventionalist wants more, namely to *replace* or *explain* this intutive understanding in terms of conventions.
- Quine's point: this seems impossible to do in a non-circular but finite way.

Structure

A Straw Man?

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This argument of Quine's has been very influential; and I think that there is no doubt that it works against its target as specified. However, it is arguable that its target as specified isn't the view that needs defeating. For, surely, it isn't compulsory to think of someone's following a rule R with respect to an expression e as consisting in his **explicitly stating** *that rule in so many words in the way that Quine's argument presupposes. On the contrary, it seems far more plausible to construe x's following rule R with respect to e as consisting in some sort of fact about x's* **behavior** *with e. (Boghossian 1996: 381)*

A Straw Man?

- Quine defeated *explicit conventionalism*, according to which all conventions must be explicitly stated.
- But that is a non-starter anyway.
- What we should be interested in is *implicit conventionalism*, on which some conventions are being followed implicitly.

A Straw Man?

Quine admits as much:

> *It may still be held that the conventions [...] are observed from the start, and that logic and mathematics thereby become conventional. It may be held that we can adopt conventions through behavior, without first announcing them in words; and that we can return and formulate our conventions verbally afterward, if we choose, when a full language is at our disposal. [...] So conceived, the conventions no longer involve us in vicious regress. (Quine 1949: 272)*

A Challenge

But he sees a problem with implicit conventions:

> *In dropping the attributes of deliberateness and explicitness from the notion of linguistic convention we risk depriving the latter of any explanatory force and reducing it to an idle label. We may wonder what one adds to the bare statement that the truths of logic and mathematics are a priori, or to the still barer behavioristic statement that they are firmly accepted, when he characterizes them as true by convention in such a sense. [...]* **as to the larger thesis that mathematics and logic proceed wholly from linguistic conventions, only further clarification can assure us that this asserts anything at all.** *(Quine 1949: 273)*

A Challenge

- If conventionalism about logic to be genuinely substantial position, it must be different from other views on logic.
- Explicit conventions would make the difference obvious.
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- If conventionalism about logic to be genuinely substantial position, it must be different from other views on logic.
- Explicit conventions would make the difference obvious.
- But is not so clear whether implicit conventions can do the trick.
- Even non-conventionalists can agree that there are implicit rules that all users of logical vocabulary follow. But there also seem to be such rules for non-logical vocabulary.
- There must be something *special* about the implicit rules of use associated with logical expressions then – otherwise implicit conventionalism becomes empty or trivial.

Towards a Solution

- It seems promising to look at the literature on *logical inferentialism*.
- General idea: meaning of logical expressions is fully determined by certain inference rules.
- We will move on to other problems next week though, and leave the force of Quine's argument undecided.

Boghossian, P. A. (1996): ‘Analyticity Reconsidered’. *Noûs*, 30(3):360–391.

Ebbs, G. (2011): ‘Carnap and Quine on Truth by Convention’. *Mind*, 120(478):193–237.

Quine, W. V. (1949): ‘Truth by Convention’. In: Feigl, H. and Sellars, W. (eds), *Reading In Philosophical Analysis*, pp. 250–273. Appleton-Century-Crofts.

Sider, T. (2011): *Writing the Book of the World*. Oxford University Press.