

# Conventionalism in Logic and Mathematics

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Lecture 1: Conventionalism: What, why, and how?

# Overview

Plan of the four lectures:

- 1 **Conventionalism: What, why, and how?**
- 2 Quine against Truth by Convention
- 3 Gödel on Convention and Consistency
- 4 Wittgenstein and Radical Conventionalism

# Structure

- 1 What is Conventionalism?
- 2 Why endorse Conventionalism?
- 3 (How) Is Conventionalism possible?

# Varieties of Conventionalism

- Many philosophical positions are called *conventionalism*.  
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- You might have encountered conventionalism about *meaning* – the view that words and sentences have meaning in virtue of speakers acting in accord with certain conventions (Lewis).
- There's also conventionalism about *modality* – the view that whether a true (for false) sentence is *necessarily* true (or false) is a matter of convention.

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This is still far from clear though.

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- Can conventions explain why (G) is true?
- To a certain extent! Consider: The sentence "Grass is green" could have meant something different from what it actually does – such as *snow is red*. (G) would then have been *false*.
- Plausibly meaning is a conventional matter. So for all statements conventions play a role in explaining their truth.

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- But obviously conventions about meaning don't *suffice* to explain why (G) is true: the world needs to be a certain way as well, namely such that grass is green.
- Upshot: In order to be interesting conventionalism must be the claim that truth can be *fully* explained in terms of conventions, for it is obvious that conventions play *some* role.

# What is Conventionalism?

## Still Rough Characterisation

The truth of logical and mathematical statements can be *fully* explained in terms of conventions.

## Complications

One influential proponent of conventionalism is Ayer:

*Like Hume, I divide all genuine propositions into two classes: those which, in his terminology, concern “relations of ideas,” and those which concern “matters of fact.” The former class comprises the a priori propositions of logic and pure mathematics, and these I allow to be necessary and certain only because they are analytic. That is, I maintain that the reason why these propositions cannot be confuted in experience is that they do not make any assertion about the empirical world, but simply **record our determination to use symbols in a certain fashion.** (Ayer 1936: 31, my emphasis)*

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- They are *true by convention*.

We will not be able to discuss all these aspects and their interrelations, and instead focus on the idea of truth in virtue of meaning.

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## Back to Logic and Math

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A sentence is true in virtue of linguistic conventions *and* the way the world is.

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Why should one diverge from this for logic and mathematics, and leave out the world completely for those cases?

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A logical truth:

The Law of Excluded Middle

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- Seems much harder to answer than in the case of "grass is green".
- An *epistemological* question: How do we know that this is true?
- Very tricky: intuitionists *reject* the law of excluded middle. Are they making a mistake? What could decide the dispute between proponents of classical and intuitionist logic?

## Back to Logic and Math

A mathematical truth:

Prime Numbers

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### Prime Numbers

There are infinitely many prime numbers.

If the truth of this is partly explained by the way the world is, then it seems that the world must contain abstract objects such as numbers in some sense. And this will seem metaphysically spooky and epistemologically dubious to many.

# Realism

Of course one could be a realist about logic and maths:

## Russell on Logic

[...] logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. (Russell 1919: 169)

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- But, so the thought goes, we know what linguistic conventions are.
- If we can explain logical and mathematical truth in terms of conventions, we explain something (apparently) mysterious in terms of something that is well understood. Progress!

## Tolerance and Pluralism



Rudolf Carnap  
*The Logical Syntax of Language* (1934/1937)

## Tolerance and Pluralism

*The fact that no attempts have been made to venture still further from the classical forms [of logic and maths] is perhaps due to the widely held opinion that any such deviation must be justified – that is, that the new language-form must be proved to be 'correct' and to constitute a faithful rendering of 'the true logic'. To eliminate this standpoint, together with the pseudo-problems and wearisome controversies which arise as a result of it, is one of the chief tasks of this book. (Carnap 1937: xiv-xv)*

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- But according to him there is no way to give a philosophical *justification* of a system of logic. We should rather allow for a plurality of logical systems, and compare them based on *pragmatic* considerations such as usefulness.
- Once this is recognised, "the boundless ocean of unlimited possibilities" lies before us (Carnap 1937: xv).

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... as we will see in the course of these lectures, 'giving syntactical rules' sounds easier than it is!

## *Empiricism, Semantics, and Ontology*

*If someone wishes to speak in his language about a new kind of entities, he has to introduce a system of new ways of speaking, subject to new rules; we shall call this procedure the construction of a linguistic **framework** for the new entities in question. And now we must distinguish two kinds of questions of existence: first, questions of the existence of certain entities of the new kind **within the framework**; we call them **internal questions**; and second, questions concerning the existence or reality **of the system of entities as a whole**, called **external questions**. (Carnap 1956: 206)*

## *Empiricism, Semantics, and Ontology*

*As an example of a system which is of a logical rather than a factual nature let us take the system of natural numbers. The framework for this system is constructed by introducing into the language new expressions with suitable rules: (1) numerals like "five" and sentence forms like "there are five books on the table";  
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*Here again there are internal questions, e.g., "Is there a prime number greater than a hundred?" **Here however the answers are found not by empirical investigation based on observations but by logical analysis based on the rules for the new expressions. Therefore the answers are here analytic, i.e., logically true.** (Carnap 1956: 208, my emphasis)*

# Why Conventionalism?

## No Mystery

Conventionalism promises to (dis)solve metaphysical and epistemological complications.

## Pluralism

Conventionalism is naturally combined with pluralism about logic and mathematics.

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## A Non-Starter?

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- Conventionalism about logic and mathematics has its attractions. It is not a very popular position, however, as there seem to be powerful arguments against it.
- Today we will look at one short but powerful influential objection against conventionalism, which I call the *master argument* against truth by convention.

## The Master Argument against Truth by Convention

*Near the middle of his book *Meaning and Modality*, Casimir Lewy takes up the theory that "necessary propositions. . . 'owe their truth to' linguistic conventions." All that conventions can do, he protests, is help to determine what a sentence says, or what proposition it expresses; whether the proposition holds true is then another question, to which rules of usage are quite irrelevant. [...] With doubtful historical accuracy I will call this the Lewy point. (Yablo 1992: 878)*

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Paul Boghossian's version of the Lewy point: If conventionalism is an interesting thesis, it must be the claim that conventions also make it true *that  $p$  is the case*, not merely that  *$S$  means  $p$* .

## Boghossian against Truth by Convention

*First, any such account would make the truth of what is expressed contingent, whereas most of the statements at stake in the present discussion are clearly necessary. Second, such an account would make the truth of the claim expressed contingent on an act of meaning, and that is very peculiar. [...]*

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*Either snow is white or it isn't*

*it wasn't the case that either snow was white or it wasn't? Isn't it overwhelmingly obvious that this claim was true before such an act of meaning, and that it would have been true even if no one had thought about it, or chosen it to be expressed by one of our sentences? (Boghossian 2017: 583)*

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Boghossian's objection: The conventionalist is committed to saying *both* are true – and that is bad.

Lewy stresses a similar point: to say that a statement is true in virtue of conventions should better not mean that it is *about* conventions.

## Replies to the Master Argument

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- If Boghossian is right, the conventionalist faces a dilemma:
- Either conventionalism entails the crazy claim that  $2+2=4$  wasn't true before any conventions were in place.
- Or it boils down to the boring claim that conventions play a role in determining logical and mathematical truth, which holds for all areas of discourse.

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- Either conventionalism entails the crazy claim that  $2+2=4$  wasn't true before any conventions were in place.
- Or it boils down to the boring claim that conventions play a role in determining logical and mathematical truth, which holds for all areas of discourse.
- The challenge: find a third notion of truth in virtue of meaning that avoids these extremes.

## Replies to the Master Argument

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- This is because the meanings of the indexical expressions are such that a speaker is guaranteed to express a true proposition.
- But at the same time these meanings do not *make* the proposition true.
- We thus seem to have a non-trivial and non-crazy notion of truth in virtue of meaning.

## Replies to the Master Argument

Gilbert Harman suggests something similar:

*[Boghossian] allows that my intention might make it the case that S means that p, but asks how that could "make it the case that S is true. Doesn't it also have to be the case that p?" The answer, in this view, is that in the first instance my intention makes it the case that S is true and in the second place that fact about my intention (is part of what) makes it the case that s means that p, where it is the case that p. This view has no commitment whatsoever as to what makes it the case that p. (Harman 1996: 394)*

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The idea seems to be that we set up linguistic conventions in such a way that certain sentences are associated with propositions that we already regard as true.

## Replies to the Master Argument

A slightly different line of defence attacks the way Boghossian's dilemma is framed. Consider again the following principle:

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This seems innocent enough. But Boghossian concludes from this that an explanation of the truth of a sentence  $S$  must be *decomposable* into two components (i) an explanation of why  $S$  means that  $p$ , and (ii) an explanation of the truth of  $p$ .

## Replies to the Master Argument

*This picture seems to assume that there are the propositions, somewhere out there all arrayed. They toil not, they spin not; they are timeless and forever. We corporeal beings work not with propositions but with sentences. Our conventions generate meaningful sentences simply by attaching them to particular propositions—like price tags at the grocery store.*

*(Warren 2015: 90f)*

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- This makes things a bit easier for the conventionalist, as it opens the possibility of truth that is fully explained by conventions without the absurd consequences Boghossian pointed out.
- Case in point: For Carnap propositions play no role in setting up linguistic conventions in the first place. They are rather a theoretical notions that is only introduced *after* we have stipulated some sentences to be true.

## Carnap on Propositions

*The system of propositions. New variables, "p," "q," etc., are introduced with a role to the effect that any (declarative) sentence may be substituted for a variable of this kind; [...]. Further, the general term "proposition" is introduced. "p is a proposition" may be defined by "p or not p" (or by any other sentence form yielding only analytic sentences) . Therefore every sentence of the form ". . . is a proposition" (where any sentence may stand in the place of the dots) is analytic. (Carnap 1956: 209f)*

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Carnap introduces talk about propositions like any other linguistic framework. He can thus deny that propositions play any substantial role in *setting up* frameworks in the first place.

## Is Conventionalism possible?

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- Next week we go into more detail by looking at Quine's famous *Truth by Convention*.

- Ayer, A. J. (1936): *Language, Truth, and Logic*. London: V. Gollancz.
- Boghossian, P. A. (2017): 'Analyticity'. In: *A Companion to the Philosophy of Language*, pp. 578–618. John Wiley & Sons, Ltd.
- Carnap, R. (1937): *The Logical Syntax of Language*. London: K. Paul, Trench, Trubner & Co.
- Carnap, R. (1956): 'Empiricism, Semantics, and Ontology'. In: *Meaning and Necessity*, pp. 205–221. University of Chicago Press.
- Harman, G. (1996): 'Analyticity Regained?' *Noûs*, 30(3):392–400.
- Russell, B. (1919): *Introduction to Mathematical Philosophy*. Dover Publications.
- Russell, G. K. (2008): *Truth in Virtue of Meaning*. Oxford University Press.
- Warren, J. (2015): 'The Possibility of Truth by Convention'. *Philosophical Quarterly*, 65(258):84–93.
- Yablo, S. (1992): 'Review of Alan Sidelle's *Necessity, Essence, and Individuation: A Defense of Conventionalism*'. *Philosophical Review*, 101(4):878.