Carnap, Beth, and the Principle of Tolerance

Benjamin Marschall

For *Rudolf Carnap Handbuch*, edited by Christian Damböck and Georg Schiemer

1. Introduction

In *The Logical Syntax of Language*, Carnap put forward his famous *Principle of Tolerance*:

> *In logic there are no morals*. Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (Carnap 1937: 52)

Understanding the Principle is essential for understanding Carnap's mature metaphilosophy. This is worth stating explicitly since the nature of Carnapian Tolerance has often been misconstrued. Putnam, for instance, took the Principle to be a thesis – roughly, the thesis that all empirically equivalent theories of the world are equally correct – that is derived from a verificationist theory of meaning (Putnam 1983: 191n). In response it has been pointed out that, for Carnap, the Tolerance is not a thesis but rather a proposal to act in a certain way. Rightly understood, the Principle expresses our practical freedom to choose between and adopt different languages (Ricketts 1994, Richardson 2007).

Carnap invokes the Principle of Tolerance to dismiss various traditional philosophical problems. One prominent example are ontological worries about the existence of abstract objects (Carnap 1956). Predictably, however, many philosophers have resisted the allure of Carnapian Tolerance, and continue to maintain that the problems he dismissed need to be addressed. The question of how a rejection of Tolerance can be motivated thus arises. Are there ways to criticise the Principle of Tolerance that – unlike Putnam's – do not rely on imposing views on Carnap that are alien to his outlook? In other words, is an *internal* critique of Carnapian Tolerance possible?

This article discusses what I consider to be the most promising candidate for such an internal critique: E. W. Beth's argument from his contribution to the Schilpp volume on Carnap (Beth 1963). Beth claims to demonstrate a "limitation regarding the Principle of Tolerance", which therefore "cannot be accepted without restrictions" (Beth 1953: 479, 499, 502). Unfortunately he did not explain the nature of this limitation particularly clearly, and so divergent interpretations are available. I will proceed as follows. First, I introduce the central thought experiment Beth relies on. I then distinguish four different ways of reading Beth's objection. The first three turn out to be ineffective against Carnap, for at times illuminating reasons. The fourth, however, might well constitute a successful internal critique of Carnap's reliance on the Principle of Tolerance.

2. *Carnap and Carnap**

Beth's thought experiment involves a character called Carnap*. Carnap* reads Carnap's *Logical Syntax* and interprets it in a way that does not accord with Carnap's intentions. Take the following enumeration, variants of which occur in the book: 0, 1, 2, 3, … (Carnap 1937: 13). How are we to understand the "…"? The natural and intended reding is that we want to talk about all and only the natural numbers. But Carnap* interprets this enumeration differently. He takes it to refer to *more* than just the natural numbers, namely to the natural numbers plus some non-standard numbers (Beth 1963: 480-481). Non-standard numbers are objects that satisfy the axioms of Peano arithmetic, and can thus be regarded as numbers, but cannot be reached by starting at 0 and adding +1 finitely many times. That such non-standard interpretations of Peano arithmetic are possible is one consequence of the Löwenheim-Skolem theorems, which inspired Beth's thought experiment (Beth 1963: 478).

The case gets more involved. In terms of modern model-theoretic semantics, it is easy to imagine Carnap and Carnap* as both interpreting one and the same formal theory with two different domains of quantification. Carnap's domain contains only the natural numbers, Carnap*'s domain has additional non-standard elements. But this cannot be all that Beth has in mind. According to him, Carnap* regards Language II – one of the languages Carnap constructs in *Logical Syntax* – as inconsistent, whereas Carnap takes it to be consistent. In order to make sense of this, one needs to assume that Carnap* uses a different *metalanguage* in which to talk *about* formal theories than Carnap (Carnap 1963: 929). In particular, Carnap*'s metalanguage needs to be such that the consistency sentence of Language II – which, using Gödelisation, expresses that Language II is consistent – is false. Beth accordingly writes that Carnap* would reject a theorem of *Logical Syntax* that shows this very consistency sentence to be true (Beth 1963: 480). The scenario is coherent because, as Gödel's second incompleteness theorem shows, the consistency sentence is independent of the axioms and rules of Language II.

3. Sharing a Metalanguage

In what sense could the case of Carnap* illustrate a limitation of the Principle of Tolerance? The first suggestion is this:

> **Reading I**
> The possibility of Carnap* shows that describing formal languages does not rule out misinterpretation. Giving formal rules is only effective if these rules are taken in the right way, which requires a shared metalanguage.

This seems to be how Carnap himself understood Beth's objection, for he writes the following in his reply:

> Since the metalanguage *ML* serves as a means of communication between author and reader or among participants in a discussion, I always presupposed […] that a fixed interpretation of *ML*, which is shared by all participants, is given. […] The necessity of this presupposition of a common interpreted metalanguage seems to me obvious. If […] I use a phrase like "no occurrence of", and a reader were to understand this phrase in the sense of "at least one

> occurrence of 'x'", then there would be no communication between us […]. (Carnap 1963: 929)

Carnap thus grants that things will go awry for readers of *Logical Syntax* with a different metalanguage. It is in principle possible to interpret "0, 1, 2, …" in an unintended way, just as someone might misinterpret the phrase "no occurrence" as meaning "at least one occurrence". But Carnap does not think that this observation demonstrates a problematic limitation of the Principle of Tolerance. And that seems the right response. For while the Principle does say that it is desirable to describe languages in terms of syntactic rules, it would be uncharitable to interpret this as entailing that the rules must be stated in a way that excludes any possibility of misinterpretation.

On the current reading, the case of Carnap* demonstrates something that should be obvious anyway: Linguistic expressions can be misunderstood by the target audience. While one can take measures to minimise the likelihood of misunderstandings, a certain degree of trust in one's audience cannot be dispensed with (Ebbs 1997: §§60-61, 2017).

4. Asymmetric Understanding

The following passage is helpful for discerning Beth's intentions:

> […] Carnap could be tolerant with respect to Carnap*, for Carnap would be able to understand why Carnap […] [rejects] certain (and indeed all) models for language II. But Carnap* would never be able to understand why Carnap […] stubbornly refuses to accept [that Language II is inconsistent] and believes Language II to have a model. (Beth 1963: 479)

It suggests another reading of the argument. The problem is not just that people using different metalanguages cannot communicate with each other, but that there are certain *asymmetries* of understanding (Friedman 1999a, b):

> **Reading II**

> In some cases Tolerance only goes one way. The one party can make sense of why the other says what they say, but not vice versa.

As before, we need to scrutinise whether this is really a limitation of Carnapian Tolerance. It would be if Carnap thought that, for every language someone uses, it needs to be possible to interpret the language of anyone else. But that is a very implausible assumption, and Carnap clearly rejects it (Goldfarb and Ricketts 1992: 69-70). In a different context he introduces an example that is structurally analogous to Beth's case. Two people, call them Less and More, have adopted different set theories, such that More accepts the existence of certain sets whose existence Less rejects. Carnap is happy to write that while More "understands both languages and the semantical rules for both", Less "understands neither [More's language] nor its semantical rules" (Carnap 1963: 873).

The asymmetry Beth flags is thus not in tension with the Principle of Tolerance as Carnap understands it. Quite the opposite, in fact. Given that there are no restrictions on which languages one can adopt, asymmetries of expressive power are just what one should expect.

5. Categoricity

Beth repeatedly refers to the "Skolem-Löwenheim paradox" (Beth 1963: 478, 483, 491), a label which alludes to some curious consequences of the Löwenheim-Skolem theorems. Take an axiomatisation of set theory such as ZFC. In ZFC, one can prove that there is an uncountable set, i.e. one whose members cannot be put into a one-to-one correspondence with the natural numbers. At the same time, it follows from the downward Löwenheim-Skolem theorem that ZFC has a countable model. It is easy to feel that there is something paradoxical about this result. One might worry that the possibility of countable interpretations shows that, in some sense, ZFC does not *really* prove that there is an uncountable set (Tymoczko 1989: 289).

Taking this analogy seriously suggests a third interpretation of Beth's argument (Ricketts 2004: 194-195):

> **Reading III**

> The possibility of Carnap* shows that formal mathematical theories do not determine a unique intended interpretation. Carnap's Language II is thus not really about the natural numbers until non-standard interpretations have been excluded.

Have we finally arrived at an internal critique of Carnapian Tolerance? Once again, the answer is no. For one thing, Carnap would reject the model-theoretic perspective on language underlying the argument, according to which it is desirable and required for a theory to 'pin down' an intended interpretation. About the case of uncountable sets, for instance, he could just say that the matter is settled once we have proved that there is an uncountable set *within* ZFC. There is no reason to think that this result can be undermined by looking at ZFC from some external perspective (Ebbs 1997: 125).

Furthermore, Carnap was well aware that formal theories are rarely ever *categorical*, meaning that all their interpretations are isomorphic to each other. Indeed, he showed that even truth-functional logic has non-standard interpretations (Carnap 1943: §§15-16). There is no indication, however, that he considered the non-categoricity of formal theories to be in tension with the Principle of Tolerance.

6. Infinitary Rule-Following

By now one may well wonder whether there is any material for an internal critique of Carnapian Tolerance to be found in Beth's paper. I think so, though the fourth reading I am about to offer is more involved than the previous three proposals. We therefore need to begin with some general reflections.

The Principle of Tolerance says that we can adopt any language we like. It might therefore seem to disable the very possibility of criticising any language someone proposes. But this tempting thought is premature. Suppose someone suggests to use a language with only logical and mathematical symbols for doing empirical science. Evidently this language is not up to the job, since physical predicates are needed. More to the point, suppose someone suggests to use a language with 578 distinct predicates. In this case one is bound to respond that we cannot adopt this language in practice, since it is too complex. On the fourth reading of Beth's argument, his point is that Carnap's Language II suffers from a similar flaw as the overly complex language just mentioned. But to see why, we need to introduce some additional details about Carnap's philosophy of mathematics.

Earlier we saw that mathematical theories are typically not categorical. Importantly, however, Carnap thought that there is a sense in which theories can be *made* categorical by means of linguistic stipulations. He thinks that we can impose linguistic rule that restrict the range of the quantifiers to the natural numbers, thus excluding the non-standard interpretation of Carnap*. One consequence of Gödel's incompleteness theorems is that no recursively axiomatizable mathematical theory with decidable inference rules is categorical. In order for Carnap's linguistic rules to do their job, they must therefore be non-recursive. One vivid example of this is the omega-rule, an inference rule with infinitely many premises (Carnap 1937: 38, 173).

Carnap's reliance on infinitary methods has been controversial. Is it appropriate to classify such non-recursive methods as linguistic rules? Many philosophers have thought not, because it is not possible to use infinitary rules in practice, unlike, say, modus ponens (though see Warren 2021). The connection to Beth's argument is that Carnap and Carnap*'s disagreement concerns the consistency sentence of Language II, whose truth value Carnap wants to settle by means of a non-recursive rule. I therefore suggest that Beth's thought experiment is supposed to illustrate how problematic this move really is:

> **Reading IV**
> One might think that Carnap's reliance on non-recursive methods is justified by the Principle of Tolerance. But – "if the language under consideration is to be used as a metalanguage" (Beth 1963: 499) – it is a non-trivial question whether a certain language can be used in practice. And Carnap's Language II fails this test, similarly to the language with 578 distinct predicates.

Whether this interpretation is what Beth actually had in mind may appear doubtful, though there are some hints in the text that speak in favour of it

(Marschall 2021: 297-298). Be that as it may, the objection from infinitary rules is inspired by Beth's Carnap* case and cannot be set aside as easily as the previous three attempts. To determine whether it is ultimately damaging or not, Carnap's philosophy of mathematics requires further scrutiny (Marschall 2022).

Carnap's mature metaphilosophy transforms many traditional philosophical problems into *pragmatic* questions about the features of certain languages (Carnap 1963: 862). The fourth reading of Beth is in line with this trend. The case of Carnap* is supposed to question the assumption that infinitary rules are usable in the same ways as finitary ones. I therefore think that we may have found the rare case of a genuinely internal challenge to Carnap's position that does not draw on assumptions alien to his philosophical outlook.

7. Conclusion

Beth's paper is intriguing but elusive. It is hard to interpret it in a coherent way without dismissing at least one of Beth's own remarks (Marschall 2021: 287, 296). The current presentation thus had to gloss over various interesting and puzzling aspects of Beth's line of reasoning. In summing up, for instance, Beth remarks that "Carnap has not been able to avoid every appeal to logical or mathematical intuitions, or, what amounts to the same, to ontological commitments" (Beth 1963: 502). Neither Carnap himself nor other commentators have been able to make much sense of the connection Beth sees between the Carnap* case and ontological questions (Carnap 1963: 933). This is not a minor point, and so important work remains to be done.

References

Beth, E. W. (1963): 'Carnap's Views on the Advantages of Constructed Systems over Natural Languages in the Philosophy of Science'. In: Schilpp, P. (ed), *The Philosophy of Rudolf Carnap*, pp. 469–502. Open Court, La Salle, Illinois.

Carnap, R. (1937): *The Logical Syntax of Language*, transl. Amethe Smeaton, La Salle: Open Court.

Carnap, R. (1939): 'Foundations of Logic and Mathematics'. In: Neurath, O., Carnap, R., and Morris, C. (eds), *International Encyclopedia of Unified Science*, pp. 139–213. University of Chicago Press.

Carnap, R. (1943): *Formalization of Logic*. Cambridge: Mass., Harvard University Press.

Carnap, R. (1956): "Empiricism, Semantics, and Ontology", in Meaning and Necessity, University of Chicago Press, 205–221.

Carnap, R. (1963). "Replies and Systematic Expositions," in *The Philosophy of Rudolf Carnap*, ed. P.A. Schilpp, La Salle: Open Court, 3–84.

Ebbs, G. (1997): *Rule-Following and Realism*. Harvard University Press.

Ebbs, G. (2017): 'Carnap's Logical Syntax'. In: *Carnap, Quine, and Putnam on Methods of Inquiry*, pp. 33–54. Cambridge University Press.

Friedman, M. (1999a): 'Analytic Truth in Carnap's Logical Syntax of Language'. In: *Reconsidering Logical Positivism*. Cambridge University Press.

Friedman, M. (1999b): 'Tolerance and Analyticity in Carnap's Philosophy of Mathematics'. In: *Reconsidering Logical Positivism*, pp. 198–233. Cambridge University Press.

Goldfarb, W. and Ricketts, T. (1992): "Carnap and the Philosophy of Mathematics," in *Wissenschaft und Subjektivität-Science and Subjectivity*, ed. David Bell and Wilhelm Vossenkuhl, Berlin: Akademie Verlag, pp. 61–78.

Marschall, B. (2021): "Carnap and Beth on the Limits of Tolerance". *Canadian Journal of Philosophy* 51 (4): 282–300.

Marschall, B. (2022): “Carnap’s Philosophy of Mathematics”. *Philosophy Compass* 17 (11): e12884.

Richardson, Alan (2007): “Carnapian Pragmatism”, in *The Cambridge Companion to Carnap*, ed. M. Friedman & R. Creath, *Cambridge University Press*, 295–315.

Ricketts, Thomas (1994): “Carnap’s Principle of Tolerance, Empiricism, and Conventionalism,” in *Reading Putnam*, ed. Peter Clark and Bob Hale, Oxford: Blackwell, 176–200

Ricketts, T. (2004): ‘Frege, Carnap, and Quine: Continuities and Discontinuities’. In: Awodey, S. and Klein, C. (eds), *Carnap Brought Home: The View from Jena*. Open Court, Chicago and LaSalle, Illinois.

Warren, J. (2021): “Infinite Reasoning”. *Philosophy and Phenomenological Research*, 103(2),